



# **”Deep Pockets”, Collateral Assignments of Patents, and the Growth of Innovations**

Bruno Amable, Jean-Bernard Chatelain, Kirsten Ralf

## **► To cite this version:**

Bruno Amable, Jean-Bernard Chatelain, Kirsten Ralf. ”Deep Pockets”, Collateral Assignments of Patents, and the Growth of Innovations. 2006. halshs-00112518

**HAL Id: halshs-00112518**

**<https://shs.hal.science/halshs-00112518>**

Preprint submitted on 8 Nov 2006

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# “Deep Pockets”, Collateral Assignments of Patents, and the Growth of Innovations

Bruno Amable\*      Jean-Bernard Chatelain†      Kirsten Ralf‡

October 3, 2006

## Abstract

This paper studies how the imperfect collateral assignments of patents contribute to “deep pockets” savings of innovative firms facing random investment opportunities in research and development (R&D) and determine the growth of their innovations, using a version of the Kiyotaki and Moore [1997] model of credit cycles. Results are: patents as collateral leverage R&D finance and magnify the effect of innovative rents on investment; the composition of current versus future financial constraints implies that firms savings decrease the steady state aggregate debt/patent ratio; the interaction between households and firms savings determines a leveraged growth of innovations which increasing when legal reforms reduce the imperfection of patents as collateral.

**Keywords:** Collateral, Patents, Research and Development, Credit rationing, Growth.

---

\*University Paris I, CES, PSE and Cepremap; 48 Boulevard Jourdan 75014 Paris France. E-mail: bruno.amable@cepremap.cnrs.fr

†University Paris X, EconomiX and Cepremap; 10 rue de Sully, 94220 Charenton le Pont, France. E-mail: jean-bernard.chatelain@u-paris10.fr

‡American University of Paris, 147 rue de Grenelle, 75007 Paris, ESLSCA, PSE and Cepremap. E-mail: Ralfkirsten@aol.com.

*“It is the uncertainty created by this legal and regulatory structure [in the United States] which lends to the very market imperfections and inefficiencies currently minimizing the ability to leverage the value of intellectual property assets and consequently stunting the economic growth of inventors and entrepreneurs”* Murphy [2002] report to the United States Patent and Trademark Office.

## 1. Introduction

The practice of lenders receiving a collateral assignment of a valuable patent portfolio conditional upon the occurrence of continuing default is slowly becoming more and more prevalent and important in the United States.<sup>1</sup> Wealth creation is increasingly based on innovation that, in turn, can give rise to important intellectual property rights. Nakamura [2003], for example, found that private US firms invested at least US Dollars 1 trillion in intangible assets in 2000, a level of investment that roughly equals the gross investment in corporate tangible assets. For many companies, these intellectual property rights may represent their most valuable assets. Unfortunately, many innovators lack the capital necessary to develop new research and must turn to outside sources for funding. Due to the limited availability of physical capital as collateral, innovators may face an external finance constraint fostered by the particular importance of adverse selection and moral hazard problems when financing R&D. There is now considerable empirical evidence that variables related to financing constraints such as leverage and/or cash flow availability are correlated with R&D investment in several countries (see Hall’s [2002] survey). Blundell, Griffith and Van Reenen [1999] explain that *“A more traditional interpretation of the innovation-market power correlation is that failures in financial markets force firms to rely on their own supra-normal profits to finance the search for innovation. The availability of internal sources of funding (‘deep pockets’) are useful for all forms of investment, but may be particularly important for R&D”*. The providers of capital may require more frequently collateral assignments of valuable patents.

Kiyotaki and Moore [1997] and Kiyotaki [1998] are seminal papers dealing with the magnifying effects of collateral availability constraints in order to explain business cycles movements. This paper applies similar incomplete financial contracts to the collateral assignment of patents combined with random profitable R&D investment opportunities. We explore their effects on firms’ R&D investment and savings at the microeconomic and at the aggregate level. Furthermore, we investigate the interaction

---

<sup>1</sup>Several services (PatentRatings, M-CAM, PLX, etc.) provide valuations of patents as collateral information since the end of the nineties (see their websites links at [www.bl.uk/collections/patents/othlinks3.html](http://www.bl.uk/collections/patents/othlinks3.html)).

between households' intermediated savings and innovators' savings and its aftermath on the growth of patents. Since innovation is viewed as a major factor of growth when monopoly rents provide incentives to entrepreneurs<sup>2</sup>, the financing of investment and the financing of R&D investment in particular may affect economic growth.<sup>3</sup>

The paper has three goals. First, it provides the conditions for significant leverage effects of the collateral assignment of patents on the growth of innovation, as this practice is likely to become widespread in the near future. It shows in particular that the dependence of innovations on past innovations increases with innovative rents *relatively more* than in standard expanding variety growth models based on R&D (Romer [1990], Grossmann and Helpman [1991] and recent contributions based on those models analyzed in Gancia and Zilibotti [2004]). Secondly, it models the joint consequences of lumpy R&D investment opportunities *and* financial constraints on individual firms savings ("deep pockets"), on the aggregate leverage (or debt/patent ratio), and on financially constrained economic growth. Finally, we derive the rate of return of innovation to be higher than the credit interest rate in a growing economy and the growth of patents to be a decreasing function of the interest rate, which is not the case in the standard R&D endogenous growth models. The model departs from the Kiyotaki and Moore [1997] small open economy credit cycle model in various ways: it is based on a closed economy with endogenous interest rate, the size of the aggregate capital stock is no longer fixed, but may grow over time, and expected monopoly rents on existing patents are used as collateral, so that they increase the value of collateral, the available amount of loans and economic growth. The model is the first one dealing with collateral assignment on patents in the economic literature on finance, innovations and growth.

Several predictions of the model are consistent with the available firm level econometric evidence. First, the reduced form cost function of R&D investment estimated by Blundell, Griffith and Van Reenen [1999] could be derived after linearization from the microeconomic model of innovators in section 2. More precisely, the novel prediction of the model is that, when valuable patents are used as collateral by an innovative firm, the sensitivity parameter of patents with respect to cash flow increases in a non linear fashion. Causality between cash flow and R&D investment goes both ways in the sense that pre-innovation rents for financially constrained firms as well as post-innovation rents are related to R&D investment (Hall, Mairesse, Branstetter and Crepon [1999]).

The model recommends an institutional policy, which has been much less advocated by economists than by lawyers (Murphy [2002]), improving the laws dealing with security interest in patents in order to greatly reduce the uncertainty surrounding the use of patents as collateral. Transfers of property rights over the income of patents

---

<sup>2</sup>See Aghion and Howitt [1992], Romer [1990], Grossman and Helpmann [1991].

<sup>3</sup>See Greenwood and Jovanovic [1990], Bencivenga and Smith [1991, 1993], King and Levine [1993], Saint-Paul [1993], Bose and Cothren [1996], De la Fuente and Marin [1996], Levine [1997], Acemoglu and Zilibotti [1997], Benhabib and Spiegel [2001], Keuschnigg [2004].

should become enforceable, not only against the debtor, but also against competing creditors (“perfection”) at low cost. Lenders have to be protected against the borrower’s ability to transfer, abandon or license the patent collateral and against the borrower’s lack of continued patent maintenance, prosecution and exploitation. These legal improvement are a way to rise the debt ceiling constraint and the growth of innovations. However, the model specifies that large effects on the growth of innovation of such an institutional policy show up only for a low equilibrium interest rate and relatively large growth rates.

The paper is organized as follows. The microeconomic behaviors of agents are described in section 2. Section 3 provides the conditions for steady state aggregate growth. Section 4 concludes the paper with a discussion of the results and related research.

## 2. The model

### 2.1. Households

A constant population of wage-earners households is distributed on  $[0, L]$ , with  $L = 1$ . On each date  $t$ , an household maximizes a constant intertemporal elasticity of substitution utility function discounted over an infinite horizon:  $U_t = \sum_{\tau=0}^{+\infty} u(c_{t+\tau}) (1 + \rho)^{-\tau}$  with  $u(c_t) = (c_t^{1-\sigma} - 1)/(1 - \sigma)$  for  $\sigma > 0$  and  $\sigma \neq 1$  or with  $u(c_t) = \ln(c_t)$  for  $\sigma = 1$ . Consumption at time  $t$  is  $c_t \geq 0$ , the rate of time preference is  $\rho \geq 0$ , the discount rate is  $1/(1 + \rho)$ , and the elasticity of substitution is  $\sigma$ . Households supply inelastically one unit of labor used in the final goods industry and are paid at a real wage rate  $w_t$ . They have no disutility of labor. They lend to entrepreneurs and earn a rate of return  $r_{t-1}$  on their wealth  $b_{t-1}^h$ . The law of motion of their wealth is:  $b_t^h = (1 + r_{t-1})b_{t-1}^h + w_t - c_t$ . The initial wealth  $b_0^h$  is given and identical for all households. Then, optimal consumption growth  $g_c$  is given by:

$$1 + g_{c,t+1} = c_{t+1}/c_t = C_{t+1}/C_t = \left( \frac{1 + r_t}{1 + \rho} \right)^{\frac{1}{\sigma}}, \quad (2.1)$$

where  $C_t = c_t L$  denotes aggregate consumption. The growth rate of consumption increases with the return on savings and decreases with the rate of time preference and the intertemporal elasticity of substitution, which measures the willingness to smooth consumption over time.

### 2.2. Production of the final good

As in other “increasing product variety” models (Romer [1990], Grossman and Helpmann [1991]), the economy has three sectors of production: a final goods sector, whose price is taken as numeraire, an intermediate goods sector, whose output is used in the production of the final good and an R&D sector which discovers blue-prints allowing

the creation of new intermediate goods. A large number of producers of the final good, indexed by  $j$ , operate in perfect competition. The final good  $Y_{jt}$  is produced from labor and intermediate inputs, which are fully used up within the period and have to be bought again the next period. Intermediate goods are defined on a set  $\{X_{jt}(i), i \in [0, N_t]\}$ . The quantity  $X_{jt}(i)$  is the amount of intermediate good  $i$  used in the production process of the final good. The value  $N_t$  represents the most recently invented intermediate good, so that the interval  $[0, N_t]$  is the variety of intermediate goods available in the economy. Technical progress is described as the invention of new intermediate goods which adds to the range of intermediate goods already invented, and implies an increase of  $N_t$  over time. Then, the constant return to scale production function of the final good for each producer is given by:

$$Y_{jt} = AL_{jt}^{1-\alpha} \int_0^{N_t} X_{jt}(i)^\alpha di \text{ with } 0 < \alpha < 1. \quad (2.2)$$

The representative producer of final goods maximizes profit while buying intermediate goods at given prices  $p_{it}$  and paying wages  $w_t$ :

$$(X_{jt}(i), L_{jt}) \in \text{ArgMax} \left( Y_{jt} - w_t L_{jt} - \int_0^{N_t} p_{it} X_{jt}(i) di \right), \quad (2.3)$$

which gives the first order conditions that marginal product has to equal the price for each input:

$$(1 - \alpha) \frac{Y_{jt}}{L_{jt}} = w_t \quad (2.4)$$

$$\alpha AL_{jt}^{1-\alpha} X_{jt}(i)^{\alpha-1} = p_{it} \text{ for } i \in [0, N_t]. \quad (2.5)$$

The last equation leads to the following demand function for intermediate inputs:

$$X_{jt}(i) = \left( \frac{\alpha A}{p_{it}} \right)^{1/(1-\alpha)} L_{jt} \text{ for } i \in [0, N_t]. \quad (2.6)$$

The demand function for intermediate inputs has a constant price elasticity:  $1/(1-\alpha) > 1$ .

### 2.3. Production of intermediate goods

The producers of each intermediate non durable good, indexed by  $i$ , act as monopolists selling their goods to the producers of final commodities at a price which adds a mark-up to marginal costs. They have to pay a rent  $\pi_{it}$  to the innovator for using her blueprint at each date  $t$ . Production of intermediate goods takes place at constant marginal cost, which is assumed to be equal to the price of final output  $Y_{jt}$  normalized

to 1. Summing the individual demand for intermediate inputs equation (2.6), the profit flow of a producer of an intermediate good is given by:

$$\max_{p_{it}} \pi_{it} = (p_{it} - 1) \sum_j X_{jt}(i) = (p_{it} - 1) \left( \frac{\alpha A}{p_{it}} \right)^{1/(1-\alpha)} \sum_j L_{jt} \quad (2.7)$$

With  $\sum_j L_{jt} = L$ . The solution for the monopoly price is

$$p_{it} = p = \frac{1}{\alpha} > 1. \quad (2.8)$$

Hence, the price  $p_{it}$  is constant over time and the same for all intermediate goods  $i$ . The monopoly price is the markup  $1/\alpha$  on the marginal cost of production equal to 1. If we substitute for  $p_{it}$  from equation (2.8) into equation (2.6), we can determine the aggregate quantity produced of each intermediate good:

$$X_t(i) = \sum_j X_{jt}(i) = X = (\alpha^2 A)^{1/(1-\alpha)} L < X^{MC} = (\alpha A)^{1/(1-\alpha)} L \quad (2.9)$$

which is also constant over time and across all intermediate goods  $i$ . Because price exceeds marginal cost, the quantity  $X$  is smaller than it would be if intermediates were priced at marginal cost  $X^{MC}$  by a factor  $\alpha^{1/(1-\alpha)} < 1$ . This ends up with a lower level of output reduced by the same factor  $\alpha^{1/(1-\alpha)}$ . If we substitute for  $p_{it}$  and  $X_t(i)$  from equation (2.8) and (2.9) into equation (2.7), we get the monopoly profit flow per intermediate good sold:

$$\pi_{it} = \pi = \left( \frac{1}{\alpha} - 1 \right) X = \left( \frac{1}{\alpha} - 1 \right) (\alpha^2 A)^{1/(1-\alpha)} L \quad (2.10)$$

which is again constant over time and across intermediate goods. The level of aggregate output is determined from equation (2.9) and (2.2) as:

$$\begin{aligned} Y_t &= \sum_j Y_{jt} = \sum_j \alpha L_{jt}^{1-\alpha} \int_0^{N_t} X_{jt}(i)^\alpha di = \alpha N_t (\alpha^2 A)^{\alpha/(1-\alpha)} \sum_j L_{jt}^{1-\alpha} L_{jt}^\alpha \\ &= \frac{1}{\alpha^2} (\alpha^2 A)^{1/(1-\alpha)} L N_t = \frac{1}{\alpha^2} X N_t < Y^{MC} = \alpha N_t (\alpha A)^{\alpha/(1-\alpha)} L \end{aligned} \quad (2.11)$$

with  $\sum_j L_{jt}^{1-\alpha} L_{jt}^\alpha = \sum_j L_{jt} = L$ . Output  $Y_t$  grows over time as the number of intermediate goods  $N_t$  expands over time due to the innovation sector. Output net of intermediate goods consumption is:

$$Y_t - X N_t = \left( \frac{1}{\alpha^2} - 1 \right) X N_t = \left( \frac{1}{\alpha} + 1 \right) \pi N_t. \quad (2.12)$$

## 2.4. R&D Sector: Technology and Finance

Every period, there is a continuum of risk neutral entrepreneurs distributed over the interval  $[0, 1]$  who are engaged in the R&D activity. They maximize the expected present value  $V_0$  of the consumption of their dividends  $d_t \geq 0$  discounted by the interest rate  $r_t$  at which they can borrow or lend, where  $E_0$  is the expectation operator at date 0:

$$V_0 = E_0 \left[ d_0 + \sum_{\tau=1}^{\tau=+\infty} \frac{d_\tau}{\Pi_{T=1}^{T=\tau} (1 + r_T)} \right] \quad (2.13)$$

They hold an initial portfolio of a number  $n_0$  of valuable blueprints and they receive an initial endowment of consumption  $d_0$ . In practice, only 5% of U.S. patents are valuable (Lemley and Shapiro [2005]). Patent protection is limited: the innovator faces the threat of obsolescence and/or opposition and litigation due to a close prior innovation and/or imitation of a proportion of her patent portfolio, with a hazard rate  $\delta$  such that the net return of innovation is assumed to remain positive:  $0 \leq \delta \leq \frac{\pi}{q} < 1$  where  $q$  is the unit cost per patent granted (Grossman and Helpman [1991], Eaton and Kortum [1999], Kwan and Lai [2003], Barro and Sala-I-Martin [2004]).<sup>4</sup> This unit technological cost per patent granted  $q$  specifies a linearized cost function of R&D investment,  $q \cdot (n_{t+1} - (1 - \delta) n_t)$ . Leading technology companies spent on average  $q = 3.8$  million U.S. dollars in R&D costs per patent granted in 1998, according to a Brody-Berman Associates survey. This lab-equipment model of expanding variety (Romer and Rivera-Batiz [1991], Barro and Sala-I-Martin [2004]) is directly related to R&D investment equations used in applied work (Blundell et al. [1999]).

The decision to invest in R&D on date  $t$  depends on two factors. First, the entrepreneur has an opportunity of finding a number of positive net present value ideas leading to new valuable patents ( $\tilde{1}_{i_t>0} = 1$ , where  $i_t$  is the number of new blueprints obtained in a period) with probability  $\theta$  ( $0 < \theta \leq 1$ ) or she has not this opportunity ( $\tilde{1}_{i_t>0} = 0$ ) with probability  $1 - \theta$ . The random variable  $\tilde{1}_{i_t>0}$  is known by the entrepreneur at the beginning of the period  $t$  and it captures the empirical observation that R&D investment is often lumpy (Geroski, Van Reenen, Walters [1997]). Second, even if she has the opportunity of obtaining valuable patents on date  $t$ , she has the opportunity to supply inelastically one unit of R&D specific human capital ( $h_t = 1$ ) in her own firm, without disutility of labour, or to withdraw her human capital ( $h_t = 0$ ), and this decision is too costly to be observed by lenders. This assumption captures the informational asymmetry between lenders and innovators. The equation of motion of the stock of blueprints of the inventor is:

$$n_{t+1} = \tilde{1}_{i_t>0} \cdot h_t \cdot i_t + (1 - \delta) n_t. \quad (2.14)$$

---

<sup>4</sup>See Barney [2002] for 1996 U.S. patents mortality rates computed over a large sample: patent life expectancy varies from 7.6 years to 18 years. Barney's table 3 presents the effects of patents life expectancy on U.S. patents average expected value and patent ratings.



A large number of risk neutral lenders or perfectly competitive financial intermediaries pool households savings in order to diversify the risks related to use of patents as collateral. An entrepreneur has always the ability to threaten its creditors by withdrawing her human capital input, consume the credit and its interest payments  $r_t b_t$  (where  $b_t$  is the stock of debt), repudiate her debt contract and find other creditors for next periods (Kiyotaki and Moore [1997]). Creditors protect themselves by collateralizing the stock of existing blueprints over which the firm has a monopoly rent and take care never to let the size of the debt repayment exceed the liquidation value  $V_{t+1}$  of the patent portfolio in the next period taking into account that lenders are not perfectly protected against the borrower's ability to transfer, abandon or license the patent collateral to a third party or to competing creditors at no legal costs (Schavey Ruff [2002] and Murphy [2002]). For a given collateralized loan, a lender faces a probability  $1 - \mu_1$  (ranging between 30% to 70% in practice) of receiving no income at all from the collateral and a probability  $\mu_1$  of receiving a random proportion  $\tilde{\mu}_2 > 0$  of the value of the collateral net of legal costs. An investor interested in lending based on such patent portfolio may reduce the risks using the law of large numbers and/or obtaining infringement enforcement insurance or defense cost reimbursement insurance from insurance companies, such as Swiss Re and Intellectual Property Insurance Services Corporation. The lender then takes into account the expected loss of the proportion  $\mu = \mu_1 E(\tilde{\mu}_2)$  of the value of the collateral  $V_{t+1}$  when deciding the amount to lend. The lender loans currently at a loan to value ratio of at most  $\frac{\mu}{1+r} = 30\%$  of the appraised value to be compared with  $\mu = 0$  at the beginning of the Nineties, where patents were not used as collateral (Edwards, [2002]).<sup>5</sup> Lenders loans according to the following debt constraint:

$$(1 + r_t) b_t \leq \mu V_{t+1} \text{ with } V_{t+1} = \pi n_t + \frac{\pi (1 - \delta) n_t}{1 + r_{t+1}} + \frac{\pi (1 - \delta)^2 n_t}{(1 + r_{t+1})(1 + r_{t+2})} + \dots \quad (2.15)$$

$$V_{t+1} = n_t \pi \left[ 1 + \sum_{\tau=1}^{+\infty} \frac{(1 - \delta)^\tau}{\prod_{T=1}^{T=\tau} (1 + r_T)} \right] \quad (2.16)$$

As in one period models, the patent portfolio  $n_t$  provides its first return  $\pi n_t$  on date  $t + 1$ . For a constant interest rate, the market value of the patent portfolio on date  $t + 1$  is:

$$V_{t+1} = n_t \pi \sum_{\tau=0}^{+\infty} \left( \frac{1 - \delta}{1 + r} \right)^\tau = n_t \pi \left( \frac{1 + r}{r + \delta} \right) \quad (2.17)$$

The credit constraint may also be written as a "leverage" or debt/patent ratio  $x_t$  bounded by an endogenous ceiling  $x^c$  (for homogeneity, the unit cost of a new patent

---

<sup>5</sup>An example is GIK Worldwide receiving a patent backed loan from Pitney Bowes Capital of 17 million USD only whereas the value of the patents in technology for delivering high speed broadcast quality video conferencing was assessed at 57 million USD: the loan to patent value is 30%.

$q$  multiplies the number of patents):

$$x_t = \frac{b_t}{qn_t} \leq x^c = \frac{\mu}{1+r_t} \frac{V_{t+1}}{qn_t} = \mu \frac{\pi}{q(r+\delta)}. \quad (2.18)$$

The collateral constraint eliminates the incentive for entrepreneurs to withdraw human capital (hence  $h_t = 1$ ) in order to gain the income  $(1+r_t)b_t$  from lenders.

## 2.5. Innovators' behaviour

The innovative firm's flow of funds constraint states that dividends should be equal to the profits at date  $t$  earned from previously discovered blueprints, to which are added new debt net of interest repayment and subtracted the cost of investment in R&D:

$$d_t = \pi n_{t-1} - q\delta n_{t-1} - r_{t-1}b_{t-1} + b_t - b_{t-1} - \tilde{1}_{i_t > 0} q(n_t - n_{t-1}) \quad (2.19)$$

On date  $t$ , entrepreneurs consume at least a positive amount of income or dividends  $d_m$  from the corporate income flow generated by previous patents, net of the accounting provision for depreciation of patents and of interest charges:

$$d_t = (1 - s_t)(\pi n_{t-1} - q\delta n_{t-1} - r_{t-1}b_{t-1}) \quad (2.20)$$

$$\geq d_m = (1 - s)(\pi n_{t-1} - q\delta n_{t-1} - r_{t-1}b_{t-1}) > 0 \quad (2.21)$$

where  $s_t < s \leq 1$  is the entrepreneurs saving rate, which is limited by  $s^6$ . Substituting consumption from the flow of funds equation (2.19) into the saving rate ceiling constraint (2.20) and using the debt ceiling constraint (2.18), one finds an upper limit on the growth of innovations determined by the ceiling of internal savings and by the debt ceiling:

$$\tilde{1}_{i_t > 0} q(n_t - n_{t-1}) \leq s \left( \left[ \frac{\pi}{q} - \delta \right] qn_{t-1} - r_{t-1}b_{t-1} \right) + x^c qn_t - b_{t-1} \quad (2.22)$$

When an innovator has an opportunity to invest ( $\tilde{1}_{i_t > 0} = 1$ ), the above inequality amounts to:

$$(1 - x^c) qn_t \leq \left( 1 + s \left( \frac{\pi}{q} - \delta \right) \right) qn_{t-1} - (1 + sr_{t-1}) b_{t-1} \quad (2.23)$$

When  $x_c \geq 1$  (that is when  $r_t < \mu \frac{\pi}{q} - \delta$ ), the above constraint does not set a ceiling on the stock of patents  $n_t$  but a negative floor. When the interest rate is such that  $\mu \frac{\pi}{q} - \delta < r_t \leq \frac{\pi}{q} - \delta$ , patents are limited by:

---

<sup>6</sup>An alternative assumption leading to similar results in this model is to assume a *logarithmic* utility of innovators consumption along with a rate of time preference  $\rho$  so that the saving rate is  $s = \frac{1}{1+\rho}$  instead of assuming a *linear* utility of consumption and a saving rate ceiling  $s$  (Kiyotaki and Moore [1997] and Kiyotaki [1998]).

$$n_t \leq \frac{\left(1 + s \left(\frac{\pi}{q} - \delta\right)\right) n_{t-1} - (1 + sr_{t-1}) \frac{b_{t-1}}{q}}{1 - x^c} \quad (2.24)$$

**Definition 1.** Let us define conditions **A1**:  $\mu \frac{\pi}{q} - \delta < r_t < \frac{\pi}{q} - \delta$ .

The entrepreneur maximizes its utility (equation 2.13) subject to the law of motions of the number of patents (equation 2.14), to the law of motion of debt (the flow of funds equation 2.19), to the debt ceiling constraint (equation 2.18) and to her saving ceiling constraint (equation 2.20). The first order condition with respect to debt is (see appendix 1 for details):

$$\lambda_t^d = \lambda_t^b + \left( \frac{1 + sr_t}{1 + r_{t+1}} \right) E_t \left( \lambda_{t+1}^d \right) \quad (2.25)$$

where  $\lambda_t^b$  is the Lagrange multiplier related to the debt/patent ceiling constraint and where  $\lambda_t^d$  is the Lagrange multiplier of the minimal consumption constraint. The minimal consumption constraint is binding ( $\lambda_t^d > 0$ ) when the entrepreneur currently faces credit constraint ( $\lambda_t^b > 0$ ) or when she expects to face a credit constraint in the future ( $E_t (\lambda_{t+k}^b) > 0$  with  $k$  a strictly positive integer). The first order condition with respect to the stock of patents is:

$$\frac{\frac{\pi}{q} - \delta - r_t}{1 + r_{t+1}} = (1 - x^c) \frac{\lambda_t^b}{1 + sE_t \lambda_{t+1}^d} + \frac{E_t \left[ \left(1 + \lambda_{t+1}^d\right) (1 - \tilde{1}_{i_{t+1}>0}) \right]}{(1 + r_{t+1}) (1 + sE_t \lambda_{t+1}^d)} \quad (2.26)$$

These two first order equations leads to three possible regimes:

**Proposition 1. Optimal R&D Investment, Saving and Borrowing at the Entrepreneur Level.**

*In each period, innovating firms can be in one of three regimes:*

(i) A perfect capital market regime is obtained when  $\lambda_t^b = E_t \lambda_{t+\tau}^d = 0$  for any integer  $\tau$ . The credit interest rate equal the marginal gain of R&D investment  $r_t = \frac{\pi}{q} - \delta$  (equation 5.6), the debt ceiling never binds, debt policy does not affect R&D investment.

Financially constrained regimes (ii) and (iii) are obtained when  $\lambda_t^d > 0$ : the entrepreneur consumes her minimal level of consumption ( $d_t = d_m$ ). Those regimes are feasible under the condition **A1** that the credit interest rate is below the marginal return of R&D investment  $r_t < \frac{\pi}{q} - \delta$  (equation 5.6). Depending on their opportunity to invest, innovators are in regime (ii) with probability  $\theta$  or in regime (iii) with probability  $1 - \theta$ .

(ii) Currently binding credit constraint regime ( $\lambda_t^b > 0$ ): The innovator has an opportunity to invest at date  $t$ . She faces binding debt and patents ceilings:

$$b_t = x^c q n_t = \frac{\mu V_{t+1}}{1 + r_t}. \quad (2.27)$$

$$n_t = \frac{\left(1 + s \left(\frac{\pi}{q} - \delta\right)\right) n_{t-1} - (1 + sr) \frac{b_{t-1}}{q}}{1 - x^c} \quad (2.28)$$

(iii) *Anticipated credit constraint regime* when  $\lambda_t^b = 0$  and  $E_t [\tilde{1}_{it+1>0}] = \theta < 1$  and  $E_t \lambda_{t+1}^d > 0$ . The innovator has no opportunity to invest and saves as much as possible in order to decrease debt, determined by the flow of funds equation (equation 2.19), as conditions A1 imply that  $0 < \frac{\pi}{q} - \delta - r < \frac{\pi}{q} - \delta - rx_{t-1}$  :

$$b_t = b_{t-1} - s \left( \left( \frac{\pi}{q} - \delta \right) q n_{t-1} - r_{t-1} b_{t-1} \right) < b_{t-1} \quad (2.29)$$

The size of the patent portfolio declines due to depreciation:

$$n_t = (1 - \delta) n_{t-1} \quad (2.30)$$

In regime (iii), if an entrepreneur has a long history of no opportunity to invest in R&D, she may eventually become a net creditor. When an entrepreneur which has built “deep pockets” over a history of no profitable ideas faces an opportunity to invest, she invests *as much as allowed by the financial constraint* due to her linear cost function and because the marginal return on R&D exceeds the credit interest rate.

### 3. Aggregate Growth of Innovations

#### 3.1. Aggregate Patent and Debt Dynamics in the Financially Constrained Regime

Given the optimal investment behavior and credit policy of firms described by proposition 1, we derive the equations of motion for the entrepreneurs’ aggregate patent and debt. Debt and patents equations are linear in patent and debt, so that aggregation across entrepreneurs does not require having to keep track of the distribution of the individual entrepreneurs patents and debt. Aggregate patents and debt are denoted by capital letters  $N_t$  and  $B_t$ . Since the population of entrepreneurs is unity, the aggregate number of patents is limited by the proportion of investing entrepreneurs ( $\theta$ ) times their patents ceilings (summing patent ceiling inequalities (2.24) over this first group of entrepreneurs) plus the proportion of non-investing entrepreneurs ( $1 - \theta$ ) times their declining stock of patents (summing equations 2.30 over this second group):

$$N_t \leq \theta \left( \frac{\left(1 + s \left(\frac{\pi}{q} - \delta\right)\right) N_{t-1} - (1 + sr_{t-1}) \frac{B_{t-1}}{q}}{1 - x^c} \right) + (1 - \theta) (1 - \delta) N_{t-1}. \quad (3.1)$$

The above inequality is an equality when condition A1 is fulfilled. One could aggregate debt across entrepreneurs in each regime using equations 2.27 and 2.29.

However, the aggregation of the flow of funds equalities (equation 2.19) leads to an equation of motion of aggregate debt where  $\theta$  is directly eliminated:

$$B_t = B_{t-1} - s \left( \left( \frac{\pi}{q} - \delta \right) q N_{t-1} - r_{t-1} B_{t-1} \right) + q N_t - q N_{t-1} \quad (3.2)$$

When  $\theta = 1$ , all firms do invest, aggregate debt is proportional to aggregate patents according to the aggregate debt ceiling constraint (equation 2.18), so that debt dynamics (3.2) is identical to patents dynamics (3.1) by a proportionality factor. The opportunity to invest or lumpiness effect ( $\theta < 1$ ) implies that the debt dynamics differs from the patent dynamics because profits are used either to decrease debt temporarily or to finance R&D investment now.

The model is closed by households aggregate consumption growth rate  $C_t = \left( \frac{1+r_{t-1}}{1+\rho} \right)^{\frac{1}{\sigma}} C_{t-1}$ , where the interest rate  $r_{t-1}$  adjusts for savings imbalances in the steady state. As households consumption  $C_t$  does not show up in entrepreneurs patents and debt dynamics, one proceeds in two steps to investigate the steady state regimes: first find R&D sector steady state aggregate debt/patent ratio (so that debt and patent grow at the same rate), then find the equilibrium interest rate such that consumption grows at the same rate as patents and debt. Using the aggregate patents equation (3.1) and the aggregate debt equation (3.2), one finds the aggregate debt/patent dynamics (cf. appendix 2):

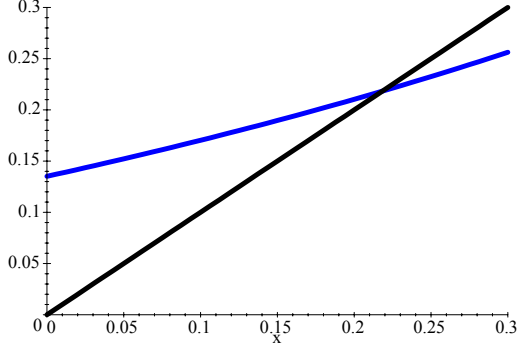
$$x_t = M(x_{t-1}) = 1 - \frac{1 - x^c}{\theta} \left( 1 - \frac{1}{1 + \left( \frac{\theta}{1-x^c} \right) \left( \frac{1+s\left(\frac{\pi}{q}-\delta\right)-(1+sr_{t-1})x_{t-1}}{(1-\theta)(1-\delta)} \right)} \right) \quad (3.3)$$

One has:

$$\frac{\partial M}{\partial \theta} > 0, \frac{\partial M}{\partial \mu} > 0, \frac{\partial M}{\partial \frac{\pi}{q}} > 0, \frac{\partial M}{\partial r_{t-1}} < 0, \frac{\partial M}{\partial x_{t-1}} > 0 \quad (3.4)$$

The steady state debt/patent ratio is such that  $x^* = M(x^*)$ .

**Example 1.** Figure 1 provides a graphical solution for the steady state loan to patent value when  $\frac{\pi}{q} = 13\%$ ,  $\delta = 8\%$ ,  $s = 60\%$ ,  $\mu = 25\%$ ,  $\theta = 50\%$ ,  $r = 3\%$ . The loan to patent value ceiling is then equal to  $x^c = 29.5\%$  (as in the GIK Worldwide case). The horizontal axis represents the debt/patent ratio  $x_{t-1}$ , and the vertical axis  $x_t$ . The line  $y = x$  intersects the increasing curve  $M(x)$  for an aggregate steady state loan to patent value  $x^* = 21.8\%$ . The aggregate loan to patent value is 7.7 percentage points below the loan to patent value ceiling because 50% of firms do not invest in R&D on a given date.



**Figure 1:** Steady State Aggregate Loan to Patent Value Ratio.

**Definition 2.** Let us define condition **A2**:  $M(0) > 0 \Leftrightarrow r_t < r_{\max} = \frac{\mu}{x_{\min}^c} \left( \frac{\pi}{q} \right) - \delta$  with  $x_{\min}^c = (1 - \theta) \frac{\frac{s\pi}{q}}{\frac{s\pi}{q} + \theta(1-\delta)}$ .

Condition A2 states that the intercept of the curve  $M(x)$  with the vertical axis is strictly positive ( $M(0) > 0$ ) so that the solution of  $x = M(x)$  is positive. A positive steady state debt  $B_t$  is obtained for an interest rate below a ceiling  $r_{\max}$  which increases with the proportion of investing entrepreneurs  $\theta$ . If assumption A2 does not hold, both aggregate groups of entrepreneurs and of households are net creditors which creates an excess supply of loanable funds in a closed economy. Condition A2 and A1 define a non empty set of feasible interest rates (as  $0 < x^c < 1$ ):

$$\mu \frac{\pi}{q} - \delta < r_t < \min \left( 1, \frac{\mu}{x_{\min}^c} \right) \cdot \frac{\pi}{q} - \delta. \quad (3.5)$$

When the loan to patent value factor rises to its current practice level  $\mu = 25\%$ , condition A2 is not likely to play a role. With the figures of the above example,  $\mu < x_{\min}^c = \frac{0.03(1-\theta)}{0.03+\theta \cdot 0.92}$ : When  $\mu = 25\%$ , the constraint A2 matters only when less than  $\theta < 8.6\%$  of innovative firms do invest in R&D (when  $\mu = 5\%$ , the constraint A2 matters only when less than  $\theta < 37.5\%$  firms do invest). Proposition 2 follows:

**Proposition 2: Financially Constrained Steady State Patent Growth for Given Interest Rate**

(i) When all firms do invest ( $\theta = 1$ ), the steady state loan to patent ratio is equal to its ceiling  $x^* = x^c$ . When some firms do not invest ( $\theta < 1$ ), under assumptions A1 and A2, a unique steady state patent and debt growth exist, with a constant strictly positive aggregate debt/patent ratio  $0 < x^* < x^c$  defined by the explicit equation (with  $R_{s,t-1} = 1 + sr_{t-1}$  and  $\Pi_s = 1 + s \left( \frac{\pi}{q} - \delta \right)$ ):

$$\begin{aligned} x^* &= \frac{1}{2\theta R} \left\{ \theta (R_s + \Pi_s) + (1 - x^c) \left[ (1 - \theta) (1 - \delta) - R_s - \sqrt{\Delta} \right] \right\} \text{ with:} \\ \Delta &= \{ [(1 - \theta) (1 - \delta) - R_s] (1 - x^c) + \theta (\Pi_s - R_s) \}^2 + 4\theta (1 - x^c) (\Pi_s - R_s) R_s \end{aligned} \quad (3.6)$$

As  $M\left(x, r_{t-1}, \frac{\pi}{q}, \theta, \mu\right) - x = 0$ , and as  $\frac{\partial M}{\partial x} < 1$  for the non demanding sufficient restriction  $r < \frac{\theta+\delta}{s}$ :

$$\frac{\partial x^*}{\partial r} = \frac{\frac{\partial M}{\partial r}}{1 - \frac{\partial M}{\partial x}} < 0, \quad \frac{\partial x^*}{\partial \frac{\pi}{q}} > 0, \quad \frac{\partial x^*}{\partial \theta} > 0, \quad \frac{\partial x^*}{\partial \mu} > 0, \quad (3.7)$$

The steady state aggregate debt/patent ratio increases with the proportion of investing firms  $\theta$ , with the loan to patent value factor  $\mu$ , with monopoly rents rewarding innovation  $\pi$  and decreases with the unit R&D investment cost  $q$  and the marginal cost of debt  $r$ .

(ii) The financially constrained (or maximal) steady state growth of aggregate patents is:

$$g_N = sr^E \text{ with } r^E = \frac{\pi}{q} - \delta + \left(\frac{\pi}{q} - \delta - r\right) \left(\frac{1}{1-x^*} - 1\right) \quad (3.8)$$

$r^E$  is the return on entrepreneurs aggregate equity and is  $\frac{x^*}{1-x^*}$  the debt/equity ratio. The growth of patents equals the growth of debt and the growth of internal equity net of the consumption of dividends (saved earnings/equity). Under the sufficient restriction  $r < \frac{\theta+\delta}{s}$ , one has:

$$\frac{\partial g_N}{\partial r} < 0, \quad \frac{\partial g_N}{\partial \frac{\pi}{q}} > 0, \quad \frac{\partial g_N}{\partial \theta} > 0, \quad \frac{\partial g_N}{\partial \mu} > 0, \quad (3.9)$$

The steady state patent growth rate increases with the proportion of investing firms  $\theta$ , with the loan to patent value factor  $\mu$ , with monopoly rents rewarding innovation  $\pi$ , and decreases with the unit R&D investment cost  $q$  and the marginal cost of debt  $r$ .

**Proof.** See appendix 2.

### 3.2. Steady State Interest Rate and Patents Growth Rate

When the interest rate is equal to the marginal return of innovation  $r_{t-1} = \frac{\pi}{q} - \delta$ , the steady state growth rate of patents is equal to:

$$g^* = g_C \left(\frac{\pi}{q} - \delta\right) = \left(\frac{1 + \frac{\pi}{q} - \delta}{1 + \rho}\right)^{1/\sigma} - 1 \approx \frac{\frac{\pi}{q} - \delta - \rho}{\sigma} \leq s \left(\frac{\pi}{q} - \delta\right)$$

It is positive as long as the interest rate is higher than the rate of time preference and which can be reached only if it is below the maximal rate of growth of patents allowed by the financial constraint when  $r_{t-1} = \frac{\pi}{q} - \delta$ , equal to  $s \cdot \left(\frac{\pi}{q} - \delta\right)$ . Else, the financially constrained growth rate may prevail under the reverse condition:

**Definition 3.** *Condition A1bis for financially constrained growth:*

$$s \cdot \left( \frac{\pi}{q} - \delta \right) < \left( \frac{1 + \frac{\pi}{q} - \delta}{1 + \rho} \right)^{1/\sigma} - 1$$

**Proposition 3: Steady State Growth Regimes**

When condition **A1bis** is not fulfilled, the credit interest rate is equal to the return of R&D investment and there exists a unique steady state growth rate  $g^* = g_C \left( \frac{\pi}{q} - \delta \right)$ .

When condition **A1bis** is fulfilled, there exists an unique equilibrium interest rate  $r^{**}$  lower the marginal return on R&D and a financially constrained growth rate  $g^{**} = g_C(r^{**}) = g_N(r^{**})$ .

**Proof.** Under condition A1bis, an equilibrium interest rate financially  $r^{**}$  determines a constrained steady state growth rate  $g^{**}$  when the growth rate of consumption equals the maximal growth rate of patents:

$$H(r) = s \left( \frac{\pi}{q} - \delta + \left( \frac{\pi}{q} - \delta - r \right) \left( \frac{1}{1 - x^*(r)} - 1 \right) \right) - \left( \frac{1+r}{1+\rho} \right)^{1/\sigma} + 1 = 0.$$

As  $H$  is a continuous and strictly decreasing function of the interest rate over the interval  $\left] \max \left( \rho, \mu \frac{\pi}{q} - \delta \right), \min \left( 1, \frac{\mu}{x_{\min}^c} \right) \frac{\pi}{q} - \delta \right]$ , as  $\lim_{r \rightarrow \mu \frac{\pi}{q} - \delta} H(r) = +\infty > 0$  and as

$H \left( \frac{\pi}{q} - \delta \right) < 0$  (condition A1bis), there exists an unique equilibrium interest rate, corresponding to an unique strictly positive patent growth rate, according to the intermediate value theorem.

**Proposition 4: Two conditions for a strong impact of Murphy's [2002] legal reforms on patents growth**

Murphy's [2002] proposes a list of legal improvements in order to greatly reduce the uncertainty surrounding the use of patents as collateral for lenders and increase  $\mu = \mu_1 E(\tilde{\mu}_2)$ . However, only under condition **A1bis**, the sensitivity of the growth of innovations with respect the loan to patent value factor  $\mu$  is positive. Then, a strong impact of a change of  $\mu$  is only obtained under the condition of a large gap between the equilibrium credit interest rate and the marginal return on innovation, that is only for high patent growth rate regimes:

$$\frac{\partial g^{**}}{\partial \mu} = \left( \frac{1 + r^{**}}{1 + \rho} \right)^{\frac{1}{\sigma} - 1} \frac{\frac{\partial g_N}{\partial \mu}}{\frac{\partial g_C}{\partial r} - \frac{\partial g_N}{\partial r}} > 0 \text{ with: } \frac{\partial g_N}{\partial \mu} = s \left( \frac{\pi}{q} - \delta - r^{**} \right) \frac{\frac{\partial x^*}{\partial x^c} \frac{\pi}{q(r^{**} + \delta)}}{(1 - x^*)^2} > 0 \quad (3.10)$$

**The collateral constraint rules out Ponzi finance problems.** As debt is always fully backed by a correct evaluation of the expected value of collateral at any



dates in the future, lenders avoid repayments problems related to Ponzi finance (see also Araujo et al. [2002]). It is not necessary to add the other no Ponzi finance condition that the growth rate has to be lower than the interest rate for equilibria. For equilibria such that the growth rate is higher than the interest rate, the entrepreneurs and households utility is infinite, as in the Ramsey problem, and transversality conditions are not necessary conditions in the infinite horizon (Barro and Sala-I-Martin [2004]). There also exist financially constrained equilibria with bounded utility as soon as the entrepreneurs maximal saving rate is below unity  $s < 1$ . One needs to control that households consumption remains positive in high growth equilibria.

**Condition for positive households consumption.** Aggregate households consumption in the steady state is obtained by the resource constraint of the economy: output is equal to households consumption, aggregate entrepreneurs consumption  $D_t$ , intermediate goods consumption and the investment in lab equipment:

$$C_t = Y_t - N_t X - q(N_{t+1} - N_t) - D_t > 0 \quad (3.11)$$

Output net of intermediate inputs  $Y_t - N_t X$  is given by equation (2.12). Hence:

$$C_t = \left(\frac{1}{\alpha} + 1\right) \pi N_t - qgN_t - D_t > 0 \Rightarrow g < \left(\frac{1}{\alpha} + 1\right) \frac{\pi}{q} - \frac{D_t}{qN_t} \quad (3.12)$$

Positive consumption are feasible for growth rate which exceed the interest rate, as  $\frac{1}{\alpha} > 1$ , (where  $\alpha$  represents the relative share of income allocated to non-labour input) and provided entrepreneurs saving rate is sufficiently high so that entrepreneurs consumption does not crowds out households consumption. To fix ideas, one denotes when there is no financial constraint, the entrepreneurs given saving rate as  $0 < s_0 \leq s$  and their debt level as a fraction  $0 < \mu_0 \leq 1$  of the debt ceiling  $B_t = \mu_0 x^* q N_t$ , so that entrepreneurs aggregate consumption is:

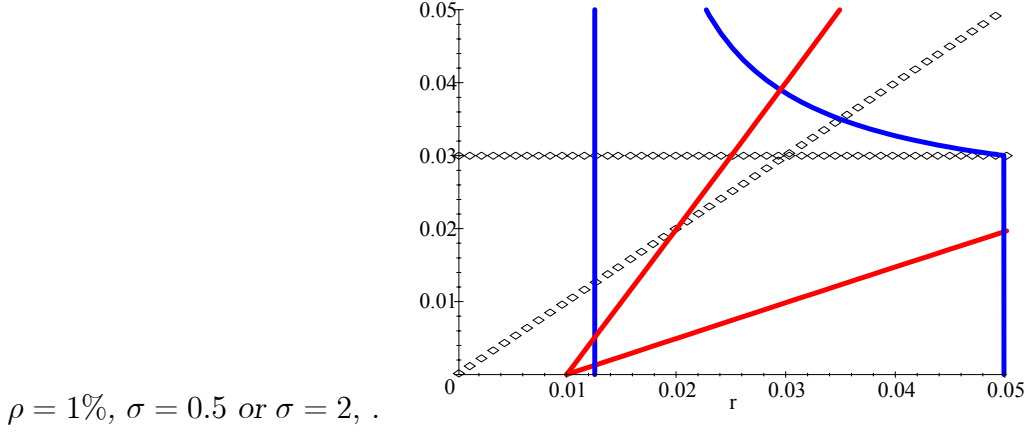
$$\begin{aligned} D_t &= (1 - s_0) ((\pi - q\delta) N_{t-1} - rB_{t-1}) \\ \frac{D_t}{qN_t} &= (1 - s_0) \left( \frac{\pi}{q} - \delta - r\mu_0 x^* \right) \frac{1}{1 + g} \end{aligned}$$

In the financially constrained regime,  $s_0 = s$  and  $\mu_0 = 1$ , so that entrepreneurs aggregate consumption is kept at its minimal level.

**No collateral regime** ( $\mu = 0$ ). Before the end of the nineties, patents were rarely used as collateral ( $\mu = 0$ ). The innovative firms are not able to use debt to finance innovation and use only their supra-normal profits to finance innovation (the "deep pocket" argument). Financially constrained patent growth is determined by  $s \left( \frac{\pi}{q} - \delta \right)$  for interest rates below  $r < \frac{\pi}{q} - \delta$ .

**Perfect collateral regime** ( $\mu = 1$ ). When the full value of patents is used as collateral, such a perfect collateral eliminates the effect of financial constraints on the growth rate of patents.

**Example 2.** Figure 2 below presents a graphical example. Parameters are set as follows:  $\frac{\pi}{q} - \delta = 5\%$ ,  $\theta = 1$ ,  $s = 60\%$ ,  $\mu = 0$  or  $\mu = 0.25$  with  $r = \mu \frac{\pi}{q} - \delta = 1.25\%$ ,



**Figure 2:** Equilibrium Growth Rates as a function of Real Interest Rates.

In figure 2, the horizontal axis represents the interest rate and the vertical axis represents the growth rate. The consumption growth curve is a line rising with the interest rate, starting from the value of the rate of time preference  $\rho = 1\%$ . The high slope rising line corresponds to  $\sigma = 0.5$ . The low slope rising line corresponds to  $\sigma = 2$ . The vertical blue line represent the asymptote of the financially constrained patent growth rate:  $r = \mu \frac{\pi}{q} - \delta = 1.25\%$  for  $\mu = 0.25$ . The patent growth curve for  $\mu = 0.25$  is first represented by a decreasing curve of interest rate as long as the growth rate is higher than  $s \left( \frac{\pi}{q} - \delta \right) = 3\%$  and the interest lower than  $r = \frac{\pi}{q} - \delta = 5\%$ . For growth rate below  $s \left( \frac{\pi}{q} - \delta \right) = 3\%$ , the patent growth rate curve is represented by a vertical line:  $r = \frac{\pi}{q} - \delta = 5\%$ , because of the free entry condition in capital markets. When no patents are used as collateral ( $\mu = 0$ ), financially constrained patent growth determines an horizontal line  $s \left( \frac{\pi}{q} - \delta \right) = 3\%$  for interest rates below  $r = \frac{\pi}{q} - \delta = 5\%$  (dark dots horizontal line). If the full value of patents could be used as collateral ( $\mu = 1$ ), the patent growth curve would be fully described by the vertical line even for growth rate over  $s \left( \frac{\pi}{q} - \delta \right) = 3\%$ , as in the perfect capital market case. Above the rising line  $g = r$ , agents' utilities are not bounded, whereas they are bounded below this line.

Let us first consider the equilibrium for a strong intertemporal elasticity of substitution:  $\sigma = 2$ . This limits the growth rate of the supply of credit. As the consequence, the equilibrium corresponds to a perfect capital market patent growth rate equal to 2% for a real interest rate equal to 5%.

Let us then consider the equilibrium for a low intertemporal elasticity of substitution:  $\sigma = 0.5$ . When patents are not used as collateral, ( $\mu = 0$ : the US in the nineties), the patent growth horizontal line intersects the consumption growth line at the growth rate:  $s \left( \frac{\pi}{q} - \delta \right) = 3\%$  for an equilibrium interest rate of 2.5%.

Now consider the case if Murphy's reforms are implemented and the practice of patents as collateral spreads across US innovative firms. Then the patent growth curve for  $\mu = 25\%$  intersect with the consumption growth curve for a 4% growth rate along with a 3% interest rate. In this case, the utility is not bounded, but for  $\sigma = 1$ , one obtains financially constrained growth rate with bounded utility.

### 3.3. Extensions

Let us consider three extensions of the model.

#### Extension 1. Endogenous Decision to Invest and R&D Growth Cycles

When the random shock affecting the marginal product of innovation ( $\eta_t \pi$ ) is not confined to two states of nature ( $\eta_t = 0$  with probability  $1 - \theta$ ,  $\eta_t = 1$  with probability  $\theta$  so that  $E_{t-1} \eta_t = \theta$ ) but distributed over a large number of states of nature according to a cumulative distribution function  $F(\varepsilon_t)$ , now with an expectation equal to unity,  $E_{t-1}(\varepsilon_t) = 1$ , then the probability of a positive net present value R&D project is given by:  $\theta = \text{Prob}(\varepsilon_t \pi \geq q(r + \delta)) = 1 - F\left(\varepsilon_t^T = \frac{q(r + \delta)}{\pi}\right)$ . The probability of investing now increases with the marginal benefit of R&D and decreases with its user cost. Then, the sensitivity of the growth rate of patents with respect to the interest rate increases by a term  $\frac{\partial g_N}{\partial \theta} \frac{\partial \theta}{\partial r}$ : this effect implies that the absolute value of the slope of the patent growth curve increases on figure 2 with respect to the case where the probability  $\theta$  is constant. However, the average profit rate  $\bar{\pi}(\theta)$  of firms who do invest depends now on the threshold value  $\varepsilon_t^T$  and on  $\theta$ : an increase of the interest rate increases the average profit rate of firms who do invest. The average  $\bar{\pi}(\theta)$  replaces  $\pi$  in the aggregate flow of funds equation (3.2) and in the aggregate patents law of motion (3.1). This second effect partially offsets the first effect of an endogenous decision to invest  $\frac{\partial g_N}{\partial \theta} \frac{\partial \theta}{\partial r}$  on the sensitivity of the growth of aggregate patents to the interest rate. One may also add macroeconomic and sectorial shocks to the idiosyncratic shock  $\varepsilon_t$  in order to generate R&D growth cycles through a time varying proportion of investing firms  $\theta_t$ .

#### Extension 2. Patent as Collateral against Open Source

There is a concern in the industry of the technology of information and of communication that patents royalties increases the cost of future research ( $\partial q / \partial \pi < 0$ ), because of the cumulative and sequential use of new ideas. There might be a growth maximizing proportion  $p_0$  of patents providing royalties and a remaining proportion  $1 - p_0$  left as "open source". This amounts to maximize the ratio  $\frac{p_0 \pi}{q(p_0 \pi)}$  for a given relation  $q(p_0 \pi)$  of the unit cost of obtaining a new patent as a function of the current cost of using existing patents for this purpose. A company such as IBM decided recently to leave as open source a proportion  $1 - p_0$  of the their patents portfolio (The Economist [2005]). However, lenders using patent portfolios as collateral will set covenants in order to limit the proportion of the patent portfolio which is left as open source. This will generate a negative correlation between the loan to collateral value factor  $\mu$  and open source (measured by  $1 - p_0$ ):  $\partial \mu / \partial p_0 > 0$ . The growth maximizing

proportion of patents not used in open source  $p_0$  will be higher when patents are used as collateral than in the case where patent as not used as collateral, because of this additional effect  $\partial\mu/\partial p_0 > 0$  in the following first order condition:

$$p_0 \partial g / \partial (\pi p_0) + (\partial g / \partial q) (\partial q / \partial p_0) + (\partial g / \partial \mu) (\partial \mu / \partial p_0) = 0$$

### **Extension 3. Collateral, Physical Capital and Leveraged Growth Miracles.**

This leveraged growth model can be extended with the addition of physical capital used as collateral. The condition that the growth rate has to be lower than the interest rate in order to avoid Ponzi finance does not fit with the observation of the long run growth level over 50 years for a dozen of postwar growth miracle countries, such as Japan and Korea, with average growth rate over 5% and average real interest rate below 5%. A high leverage for leading large firms, along with low real interest rates and low dividends, using loans backed by collateral on physical assets and by government insurance is a necessary technique to promote high speed growth although it is not sufficient to explain the efficient allocation of capital in those growth miracles. Contrary to conventional wisdom, financial constraints are not necessarily incompatible with high speed growth.

## **4. Conclusion**

This paper describes an endogenous growth model with lenders limiting credit up to the collateralizable value of existing patents and with a composition between innovative firms facing a probability to find a positive net present value R&D investment opportunity or not each period.

First, at the entrepreneur level, financial constraints and lumpiness lead to a specific entrepreneurs savings behaviour where they build “deep pockets” by anticipating future financial constraints. When a lumpy R&D investment opportunity occurs, the dependance of the persistence of R&D investment on the markup rewarding innovations is amplified by the debt/patent collateral constraint.

Secondly, the aggregation of entrepreneurs behaviour, some of them saving for future investment, some of them currently investing, determines a steady state endogenous aggregate leverage (or debt/patent ratio) below the leverage ceiling.

Thirdly, this financially constrained steady state occurs only for relatively large growth rates. In these regime, a large effect on growth of reforms protecting lenders using patents as collateral occurs for low values of the equilibrium interest rate with respect to the rate of return on innovation, a factor which depends also on the growth of credit supply and not only on the behaviour of innovative firms.

Extensions suggested that collateral assignment of patents may be detrimental to open source, because it adds incentives to value patents portfolios and that leverage driven growth is a necessary characteristic of high speed growth.

## References

- [1] Acemoglu D. and Zilibotti F. [1997] "Was Prometheus Unbound by Chance? Risk Diversification and Growth." *Journal of Political Economy*, 105, pp.709-51.
- [2] Aghion P., Howitt P. [1992]. A Model of Growth through Creative Destruction". *Econometrica* 60, 323-51.
- [3] Araujo A., Pascoa M.R. and Torres-Martinez P. [2002]. Collateral Avoids Ponzi Schemes in Incomplete Markets, *Econometrica*, 70(4), pp. 1613-1637.
- [4] Barro R. and Sala-I-Martin [2004]. *Economic Growth* (2nd Edition). Mac Graw Hill.
- [5] Barney J.A. [2002]. A Study of Patent Mortality Rates: Using Statistical Survival Analysis to Rate and Value Patent Assets. *American Intellectual Property Law Association Quarterly Journal*. 30(3), p. 317-352.
- [6] Bencivenga V. and Smith B. [1991] Financial Intermediation and Endogenous Growth. *Review of Economic Studies*. **58**, 195-209.
- [7] Bencivenga V. and Smith B. [1993] Some Consequences of Credit Rationing in an Endogenous Growth Model. *Journal of Economic Dynamics and Control*. **17**, 97-122.
- [8] Benhabib J. and Spiegel M.M. [2001] The Role of Financial Development in Growth and Investment. *Journal of Economic Growth*. **5(4)**, 341-360.
- [9] Blundell R., Griffith R. and Van Reenen J. [1999]. Market Share, Market Value and Innovation in a Panel of British Manufacturing Firms. *Review of Economic Studies*, vol. 66, pp. 529-554.
- [10] Bose N. and Cothren R. [1996]. Equilibrium Loan Contracts and Endogenous Growth in the Presence of Asymmetric Information. *Journal of Monetary Economics*. **38**, 363-376.
- [11] De la Fuente A. and Marin J.M. [1996]. Innovation, Bank Monitoring and Endogenous Financial Development. *Journal of Monetary Economics*. **38**, 269-301.
- [12] Eaton J. and Kortum S. [1999]. International Technology Diffusion: Theory and Measurement. *International Economic Review*. **40**, 537-570.
- [13] Edwards D. [2002]. Patent Backed Securization: Blueprint for a New Asset Class. [www.surfi.gov.sg/sip/site/focus/surfi\\_focus\\_052002.htm](http://www.surfi.gov.sg/sip/site/focus/surfi_focus_052002.htm).
- [14] *The Economist* [2005]. A Market of Ideas: A Survey of Patents and Technology. 22nd october.

- [15] Gancia G. and Zilibotti F. [2004]. Endogenous Growth with Product Variety, forthcoming in *the Handbook of Economic Growth* (P. Aghion and S. Durlauf, eds.).
- [16] Geroski, P., J. Van Reenen and C.F. Walters [1997]. "How persistently do firms innovate?" , *Research Policy*, 26, 33-48.
- [17] Greenwood J. and Jovanovic B. [1990] Financial Development, Growth, and the Distribution of Income. *Journal of Political Economy*. **98(5)**, 1076-1107.
- [18] Grossman, G.M. and Helpman, E. [1991] *Innovation and Growth in the Global Economy*. The MIT Press, Cambridge, MA.
- [19] Hall B.H. [2002] The Financing of Research and Development. *Oxford Review of Economic Policy*. 18(1), 35-51.
- [20] Hall B.H., Mairesse J., Branstetter L., and Crepon B. [1999]. "Does Cash Flow Cause Investment and R&D: An Exploration Using Panel Data for French, Japanese and United States Scientific Firms". In Audretsch, D., and A. R. Thurik (eds.), *Innovation, Industry Evolution, and Employment*, Cambridge, England: Cambridge University Press.
- [21] Keuschnigg C. [2004]. Venture Capital Backed Growth. *Journal of Economic Growth*. **9**, 239-261.
- [22] King R. and Levine R. [1993]. Finance, Entrepreneurship, and Growth: Theory and Evidence. *Journal of Monetary Economics*. **32(3)**, 513-542.
- [23] Kiyotaki N. and Moore J. [1997]. Credit Cycles. *Journal of Political Economy*. **105(2)**, 211-248.
- [24] Kiyotaki N. [1998]. Credit and Business Cycles, *Japanese Economic Review*. **49(1)**, 18-35.
- [25] Kwan F.Y.K and Lai E.L.C. [2003]. Intellectual Property Rights Protection and Endogenous Economic Growth. *Journal of Economic Dynamics and Control*. **27**, 853-873.
- [26] Lemley M.A. and Shapiro C. [2005]. Probabilistic Patents. *Journal of Economic Perspectives*, 19(2), pp.75-98(24)
- [27] Levine R. [1997]. Financial Development and Economic Growth: Views and Agenda. *Journal of Economic Literature*. **25**, 688-726.
- [28] Murphy W.J. [2002]. A Proposal for a Centralized and Integrated Registry for Security Interests in Intellectual Property. *IDEA The Journal of Law and Technology*. Franklin Pierce Law Center, Concord, USA. 41(3-4), pp. 297-601 (41 IDEA 297).

- [29] Nakamura L. [2003]. A trillion dollars a year in intangible investment and the new economy. In *Intangible Assets: Values, Measures, and Risks*, (J. Hand and B. Lev editors.), pp. 19-47. Oxford University Pres, London.
- [30] Rivera-Batiz L. and Romer P. [1991]. Economic Integration and Endogenous Growth. *Quarterly Journal of Economics*. **106**. 531-55.
- [31] Romer, P. [1990] Endogenous Technological Change. *Journal of Political Economy*. **98(5)** pt. 2, S71- S102.
- [32] Saint-Paul G. [1992] Technological Choice, Financial Markets and Economic Development. *European Economic Review*. **36**, 763-781.
- [33] Schavey Ruff D. [2003]. Security Interest in Patents and Trademarks. *Mayer, Brown, Rowe and Maw LLP Publication*, [http://www.securitization.net/pdf/Ruff\\_chavey\\_security\\_interests.pdf](http://www.securitization.net/pdf/Ruff_chavey_security_interests.pdf).

## 5. Appendix 1

The Lagrangian of the entrepreneur program is:

$$(n_t, b_t) \in \text{Arg max } E_0 \sum_{t=1}^{+\infty} \frac{L_t}{(1+r)^t} \quad (5.1)$$

$$\text{with } L_t = (1 + \lambda_t^d) d_t + \lambda_t^b (qx^c n_t - b_t) + \lambda_t^d (-(1-s)([\pi - q\delta] n_{t-1} - r_{t-1} b_{t-1})) \quad (5.2)$$

where  $\lambda_t^b$  is the Lagrange multiplier related to the debt ceiling constraint,  $\lambda_t^d$  is the Lagrange multiplier related to the minimal consumption constraint, and with consumption  $d_t$  given by the flow of funds constraint:

$$d_t = (\pi - q\delta) n_{t-1} + b_t - (1 + r_{t-1}) b_{t-1} - \tilde{1}_{i_t > 0} q (n_t - n_{t-1}) \quad (5.3)$$

The Euler equation on debt  $b_t$  is  $\frac{\partial L_t}{\partial b_t} = 0$ , for any date  $t$ :

$$0 = 1 + \lambda_t^d - \lambda_t^b + \frac{E_t \left( -(1+r_t) \left( (1 + \lambda_{t+1}^d) + \lambda_{t+1}^d (1-s) r_t \right) \right)}{1 + r_{t+1}} = 0 \Rightarrow \quad (5.4)$$

$$\lambda_t^d = \lambda_t^b + \left( 1 - \frac{(1-s)r_t}{1 + r_{t+1}} \right) E_t (\lambda_{t+1}^d) = \lambda_t^b + \left( \frac{1 + sr_t}{1 + r_{t+1}} \right) E_t (\lambda_{t+1}^d) \quad (5.5)$$

The first order condition with respect to the stock of patents is  $\frac{\partial L_t}{\partial n_t} = 0$ , that is:

$$0 = (1 + \lambda_t^d) (-q) + qx^c \lambda_t^b + E_t \left( \frac{1}{1 + r_{t+1}} \right) \left( (1 + \lambda_{t+1}^d) (\pi - q\delta - \tilde{1}_{i_{t+1} > 0} q) + \lambda_{t+1}^d (-(1-s)(\pi - q\delta)) \right)$$

One divides by  $q$  and substitutes  $\lambda_t^d$  using the first order condition for debt:

$$0 = -1 - \lambda_t^b - \left( \frac{1 + sr_t}{1 + r_{t+1}} \right) E_t \left( \lambda_{t+1}^d \right) + x^c \lambda_t^b \\ + E_t \left( \frac{1}{1 + r_{t+1}} \right) \left( \left( 1 + \lambda_{t+1}^d \right) (1 - \tilde{1}_{i_{t+1} > 0}) + 1 + \frac{\pi}{q} - \delta + \lambda_{t+1}^d \left( 1 + s \left[ \frac{\pi}{q} - \delta \right] \right) \right)$$

Hence:

$$\frac{\frac{\pi}{q} - (r_t + \delta)}{1 + r_{t+1}} = (1 - x^c) \frac{\lambda_t^b}{1 + s E_t \lambda_{t+1}^d} + \frac{E_t \left[ \left( 1 + \lambda_{t+1}^d \right) (1 - \tilde{1}_{i_{t+1} > 0}) \right]}{(1 + r_{t+1}) \left( 1 + s E_t \lambda_{t+1}^d \right)} \quad (5.6)$$

## 6. Appendix 2

The law of motion of the debt/patent ratio  $x_t$  as a function of its previous value:  $x_t = M(x_{t-1})$  is computed using the aggregate patent growth factor:

$$\frac{N_t}{N_{t-1}} = g_\theta = \theta \left( \frac{\Pi_s - R_{s,t-1} x_{t-1}}{1 - x^c} \right) + (1 - \theta) (1 - \delta). \quad (6.1)$$

The flow of funds equation can be written as:

$$\frac{N_t}{N_{t-1}} = g_F = \frac{\Pi_s - R_{s,t-1} x_{t-1}}{1 - x_t} = \Pi_s + \left( \Pi_s - R_{s,t-1} \frac{x_{t-1}}{x_t} \right) \left( \frac{1}{1 - x_t} - 1 \right). \quad (6.2)$$

This leads to the implicit relation  $N(x_t, x_{t-1}) = 0$ :

$$N(x_t, x_{t-1}) = g_F - g_\theta = \frac{\Pi_s - R_{s,t-1} x_{t-1}}{1 - x_t} - \theta \left( \frac{\Pi_s - R_{s,t-1} x_{t-1}}{1 - x^c} \right) - (1 - \theta) (1 - \delta) = 0. \quad (6.3)$$

which can be written as this explicit equation:

$$x_t = M(x_{t-1}, r_{t-1}) = 1 - \frac{\Pi_s - R_{s,t-1} x_{t-1}}{\theta \left( \frac{\Pi_s - R_{s,t-1} x_{t-1}}{1 - x^c} \right) + (1 - \theta) (1 - \delta)} \quad (6.4)$$

$$= 1 - \frac{1 - x^c}{\theta} \left( 1 - \frac{1}{1 + \left( \frac{\theta}{1 - x^c} \right) \left( \frac{\Pi_s - (1 + sr_{t-1}) x_{t-1}}{(1 - \theta)(1 - \delta)} \right)} \right) \quad (6.5)$$

A sufficient condition for  $\frac{\partial M}{\partial x_{t-1}} < 1$ :

$$\frac{\partial M}{\partial x_{t-1}} = \frac{(1 + sr_{t-1}) (1 - \theta) (1 - \delta)}{\left[ \theta \left( \frac{\Pi_s - R_{s,t-1} x_{t-1}}{1 - x^c} \right) + (1 - \theta) (1 - \delta) \right]^2} < 1 \quad (6.6)$$

$$r_{t-1} < \frac{\theta + \delta}{s} < \frac{(1 + g_\theta)^2}{(1 - \theta) (1 - \delta)} - 1 \Rightarrow \frac{\partial M}{\partial x_{t-1}} < 1 \quad (6.7)$$



The steady state debt/patent ratio  $x$  is given by the following quadratic equation:

$$N(x, x) = R_s + \frac{1 + s\left(\frac{\pi}{q} - \delta\right) - R_s}{1 - x} - \theta \left( \frac{1 + s\left(\frac{\pi}{q} - \delta\right) - R_s x}{1 - x^c} \right) - (1 - \theta)(1 - \delta) = 0$$

The function  $N(x, x)$  is continuous on the interval  $[0, x^c]$  and strictly increasing:

$$\frac{\partial N(x, x)}{\partial x} = \frac{1 + s\left(\frac{\pi}{q} - \delta\right) - R_s}{(1 - x)^2} + \theta \frac{R_s}{1 - x^c} > 0$$

According to the intermediate value theorem, a unique solution exist for a positive steady state debt/patent ratio  $0 < x^* \leq x^c < 1$  under the conditions  $N(0) < 0$  and  $N(x^c) > 0$ . First,  $N(x^c) > 0$  is always fulfilled as long as  $\theta \leq 1$ :

$$N(x^c, x^c) = R_s + \frac{1 + s\left(\frac{\pi}{q} - \delta\right) - R_s}{1 - x^c} - (1 - \theta)(1 - \delta) - \theta \frac{1 + s\left(\frac{\pi}{q} - \delta\right) - x^c R_s}{1 - x^c} \geq 0$$

Second,  $N(0, 0) < 0$  leads to condition A2:

$$\begin{aligned} N(0, 0) &= 1 + s\left(\frac{\pi}{q} - \delta\right) - (1 - \theta)(1 - \delta) - \theta \frac{1 + s\left(\frac{\pi}{q} - \delta\right)}{1 - x^c} < 0 \\ \Rightarrow x^c &= \frac{\mu \frac{\pi}{q}}{r + \delta} > x_{\min}^c = (1 - \theta) \frac{\frac{s\pi}{q}}{\frac{s\pi}{q} + \theta(1 - \delta)} \end{aligned}$$

Condition A2 for the steady state debt/patent ratio to be strictly positive implies that the interest rate should be below the ceiling  $r_{\max}$ :

$$r < r_{\max} = \frac{\mu}{x_{\min}^c} \frac{\pi}{q} - \delta \quad (\text{A2})$$

The explicit solution  $x^*$  is found by solving the quadratic equation  $N(x, x) = 0$ .

## 7. Appendix 3. Transitory Dynamics following Legal Reforms

In the perfect capital market case or in the financially constrained case without lumpiness effects ( $\theta = 1$ ), there are no transitory dynamics on aggregate variables following an increase of the loan to patents used as collateral factor  $\mu$  after the legal reform. Hence, the economy jumps from the old to the new steady state when there is a change of exogenous parameters of the model (in the example 2: from the equilibrium with  $\mu = 0$  to the equilibrium with  $\mu = 25\%$ , from a 3% growth rate to a 4% growth rate). The fact that some firms are not able to invest ( $\theta < 1$ ) and save instead of investing introduces some sand and persistence in the aggregate debt dynamics ( $x_t = M(x_{t-1}, r_{t-1})$ ), which is the origin of transitory dynamics.

The growth rate of aggregate debt of entrepreneurs (determined by the flow of funds equation) has to match the growth rate of households savings (identical to the growth rate of households consumption) at each period  $t$  in order to correct savings imbalances during the transitory dynamics. The interest  $r_{t-1}$  is derived as follows:

$$\frac{C_t}{C_{t-1}} = \left( \frac{1 + r_{t-1}}{1 + \rho} \right)^{\frac{1}{\sigma}} = \frac{\Pi_s - (1 + sr_{t-1})x_{t-1}}{1 - x_t} = \frac{\Pi_s - (1 + sr_{t-1})x_{t-1}}{1 - M(x_{t-1}, r_{t-1})} \quad (7.1)$$

Knowing  $r_{t-1}$ , one proceeds to the next step using  $x_t = M(x_{t-1}, r_{t-1})$  and solving the above equation for date  $t+1$  in order to find  $r_t$ .<sup>7</sup> The convergence will be such that  $x^*(\mu = 0) = 0 < x_{t-1} < x_t \leq x^*(\mu = 25\%)$  with  $r^*(\mu = 0) = 2.25\% < r_{t-1} < r_t \leq r^*(\mu = 25\%) = 3\%$ . The slope of the function  $M$  ( $0 < M'(x_t) < 1$ ) ensures regular convergence with to this steady state, without any cyclical, chaotic or indeterminate patterns (cf. figure 1). During the transition, the growth rate of patents, which depends on  $\theta$ , is lower than the growth rate of debt of entrepreneurs when  $x_t < x_{t+1}$  and can be computed using the last dynamical equation of the model (with  $r_{t-1}$  given by the previous computation):

$$N_t = 1 - \delta + \theta \left( \frac{1 + s \left( \frac{\pi}{q} - \delta \right) - (1 + sr_{t-1})x_{t-1}}{1 - x^c} - (1 - \delta) \right) N_{t-1}. \quad (7.2)$$

When the interest rate rises, so does the economy growth rate, as in other convergence models based on Ramsey's savings behaviour. Nonetheless, these dynamics present a remarkable feature. Because of financial constraints, the transitory dynamics of the credit interest rate and of the marginal productivity is no longer predicted to be identical. This is not the case in the Ramsey-Solow model of GDP per head convergence of nations, based on perfect capital markets, where the user cost equals the marginal product of capital.

---

<sup>7</sup>For the sake of simplicity, it is assumed here that the forward looking debt ceiling variable  $x^c$  discounts patents royalties at the final steady state interest rate  $r^{**}$  after the initial shock. This means that the debt ceiling is lower than when discounting the future royalties of existing patents by the future sequence of transitory interest rate (a second order effect which increases marginally the speed of convergence).